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Comparing the redshift-space density field to the real-space velocity field

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ABSTRACT
I derive a nonlinear local relation between the *redshift-space* density field and the *real-space* velocity field. The relation accounts for radial character of redshift distortions, and it is not restricted to the limit of the distant observer. Direct comparisons between the observed redshift-space density fields and the real-space velocity fields possess all of the advantages of the conventional redshift-space analyses, while at the same time they are free of their disadvantages. In particular, neither the model-dependent reconstruction of the density field in real space is necessary, nor is the reconstruction of the nonlinear velocity field in redshift space, questionable because of its vorticity at second order. The nonlinear redshift-space velocity field is irrotational only in the distant observer limit, and that limit is not a good approximation for shallow catalogs of peculiar velocities currently available. Unlike the conventional redshift-space comparisons, the comparison proposed here does not have to be restricted to the linear regime. Accounting for nonlinear effects removes one of the sources of bias in the estimate of β. Moreover, the nonlinear effects break the Omega–bias degeneracy plaguing all analyses based on linear theory.

Key words: cosmology: theory, dark matter, large-scale structure of the Universe

1 INTRODUCTION

Comparisons between density fields of galaxies and the fields of their peculiar velocities are a powerful tool to measure the cosmological parameter Ω. This is because in the gravitational instability paradigm the density and the velocity fields are tightly related, and the relation between them depends on Ω.

These comparisons are commonly performed in real space. A necessary ingredient of real-space analyses is the reconstruction of the galaxy density field in real space from the observed galaxy field, which is given only in redshift space. While performing such a reconstruction, one has to correct the redshifts for peculiar velocities of galaxies. However, the amplitude of these velocities depends on Ω, or in the case of bias, on β. This is a serious drawback of real-space comparisons: they necessarily assume the value of a parameter which is to be subsequently estimated.

To avoid this problem, the comparisons in redshift space have been proposed (Nusser & Davis 1994, hereafter ND). However, the velocity field in redshift space is irrotational only at the linear order,* so that the redshift-space analyses must be restricted to the linear regime. This is unsatisfactory, since the derived amplitude of the density fluctuations from current redshift surveys (e.g., Fisher et al. 1995) and from the POTENT reconstruction of density fields (Dekel et al. 1990 and Bertschinger et al. 1990), slightly exceeds the linear regime. For example, the density contrast in regions like the Great Attractor or Perseus-Pisces is about unity even when smoothed over scales of 1200 km s^{-1} (Sigad et al. 1998). Future redshift surveys and peculiar velocity catalogs are expected to provide reliable estimates of density and velocity fields on scales where nonlinear effects are certainly non-negligible. Those effects may lead to interesting consequences, such as breaking the degeneracy between Ω and bias (Dekel et al. 1993, Bernardeau et al. 1999, Chodorowski 1999; hereafter C99), and therefore they should be accounted for.

* Chodorowski & Nusser (1999) have shown that the nonlinear redshift-space velocity field is irrotational in the distant observer limit. However, the catalogs of peculiar velocities are not yet deep enough for this assumption to be well satisfied (Zaroubi & Hoffman 1996, Szalay, Matsubara & Landy 1998).

Here I propose to compare the *redshift-space* density field directly to the *real-space* velocity field. Such a comparison enables one to avoid the model-dependent reconstruction of the density field in real space. Also, the vorticity of the velocity field is no longer a problem, because (before shell crossings) the real-space velocity field is irrotational without any restrictions. Therefore, the comparison proposed here combines all advantages of real-space and redshift-space analyses, while at the same time it is free from their disadvantages.

The present analysis is based on the work of C99, who derived a similar relation in the distant observer limit (DOL). However, as pointed out first by Zaroubi & Hoffman (1996), and more recently by Szalay, Matsubara & Landy (1998), the DOL approximation is not well satisfied by available catalogs of galaxy redshifts and peculiar velocities. Here I relax this assumption, and I take fully into account the radial character of redshift distortions. I show how the derived relation can be inverted, i.e., how to reconstruct the real-space velocity field from the redshift-space density field. This is a necessary ingredient of the so-called velocity–velocity comparisons. The results of the paper are then compared to the results of ND.

The outline of this paper is as follows: in Section 2 I derive the exact relation between the redshift-space density and the real-space velocity. In Section 3 I expand it explicitly up to linear and second-order terms. Next, in Section 4 I derive a similar relation between the *galaxy* density field and the velocity field under an assumption of a nonlinear but local bias. In Section 5 I show how such a relation can be used to break the Ω–bias degeneracy in velocity–velocity comparisons. Summary and conclusions are in Section 6.

2 EXACT RELATION

The transformation from the real space coordinate, $\boldsymbol{r}$, to the redshift space coordinate, $\boldsymbol{s}$, is (Kaiser 1987; hereafter K87)

$$\boldsymbol{s} = \boldsymbol{r} + v_r \hat{\boldsymbol{r}} \,, \tag{1}$$

where $v_r(\boldsymbol{r}) = \boldsymbol{v} \cdot \hat{\boldsymbol{r}}$; $\boldsymbol{v}(\boldsymbol{r})$ is the real-space velocity field, and velocities are measured relative to the Local Group. The relation between the redshift-space mass density field, δ_s, and the real-space velocity field can be obtained from the equation

$$\delta_s(\boldsymbol{s}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left\{ \frac{1}{r^2} \frac{\partial^n}{\partial r^n} \left[r^2 v_r^n \left(\frac{\phi(r)}{\phi[s(r)]} (\delta + 1) - J \right) \right] \right\} \Bigg|_{\tilde{\boldsymbol{r}} = \boldsymbol{s}} . \tag{2}$$

Here, ϕ is the selection function, $\delta(\boldsymbol{r})$ is the real-space density field, and $J(\boldsymbol{r})$ is the Jacobian of the transformation (1),

$$J(\boldsymbol{r}) = (1 + v_r/r)^2 (1 + v_r') \,, \tag{3}$$

where $' \equiv \partial/\partial r$.

Equation (2) follows from the continuity equation; its derivation is outlined in Appendix A. The equation is exact; however it is strictly valid only in the absence of triple-value zones in redshift space. It is a generalization of equation (A6) of C99, which was derived in the DOL.

The relation between the redshift-space density and the real-space velocity follows from equation (2) when the density–velocity relation (DVR) in real space is used. The DVR in real space is well known, it has been extensively studied both analytically and numerically (Nusser et al. 1991, Bernardeau 1992, Gramann 1993, Mancinelli et al. 1994, Mancinelli & Yahil 1995, Chodorowski 1997, Chodorowski & Łokas 1997, Chodorowski et al. 1998, Bernardeau et al. 1999, Ganon et al. 2000, Kudlicki et al. 2000). While a number of accurate formulas has been proposed, for the purpose of this Section it is sufficient that the real-space density is a well-known nonlinear function of the first-order velocity derivatives, with an explicit dependence on Ω,

$$\delta(\boldsymbol{r}) = \mathcal{F}[\partial v_i/\partial r_j(\boldsymbol{r}), \Omega] \,. \tag{4}$$

If the selection function is given by a power law, $\phi \propto s^{-p}$, then

$$\frac{\phi(r)}{\phi[s(r)]} = (1 + v_r/r)^p \,. \tag{5}$$

If not, the expression $\phi(r)/\phi(s)$ should be expanded explicitly. In any case, it will be a function of v_r/r. Therefore, both $\phi(r)/\phi(s)$, δ and J are functions of the real-space velocity field and its derivatives, and equation (2) becomes a relation between the redshift-space density and the real-space velocity.

The key point of equation (2) is that it relates the redshift-space density at a point $\boldsymbol{s}$ to an expression involving the real-space velocity field evaluated at $\tilde{\boldsymbol{r}} = \boldsymbol{s}$. ($\tilde{\boldsymbol{r}}$ is a real-space point, in general different from $\boldsymbol{r}$, which is related to $\boldsymbol{s}$ by eq. 1.) This has been achieved by effectively expanding real-space quantities, appearing in the continuity equation as functions of $\boldsymbol{r}$, around $\tilde{\boldsymbol{r}} = \boldsymbol{s}$ (for details see C99 and Appendix A). This is very convenient, since now we can treat the two fields as if they were given in the same coordinate system. In other words, we can compare the redshift-space density at a redshift, say, 3000 km s^{-1}, directly to the real-space velocity at a real-space position 3000 km s^{-1}.

Relation (2) involves an infinite series in velocity. If the density–velocity comparison is performed on scales large enough so that they are only weakly nonlinear, then this series can be truncated. Truncation at order q means that the terms of the order $q+2$ in velocity or higher are neglected. The truncation procedure should be done with care, because nonlinear effects are stronger in redshift space than in real space (e.g., Taylor & Hamilton 1996, Scoccimarro, Couchman & Frieman 1998, this paper). For example, the formalism presented here fails for triple-value zones (regions with shell crossings in redshift space). However, these regions or structures in real space may be just after turning around, quite a mildly nonlinear condition. Thus far, all density–velocity comparisons in redshift space have been restricted to linear regime (e.g., ND, Fisher et al. 1995). In the next section I will derive an explicit expression for the relation (2) up to second-order terms. Moreover, it will become clear how to extend the derivation to still higher order, if necessary.

3 APPROXIMATE RELATION

3.1 Linear relation

To derive the linear relation, we linearize the ratio of the selection functions, the real-space DVR, and the Jacobian. We have

$$\frac{\phi(r)}{\phi[s(r)]} \simeq 1 - \frac{d\ln\phi}{d\ln r}\frac{v_r}{r}\,, \tag{6}$$

$$\delta \simeq -f^{-1}\nabla_{\boldsymbol{r}}\cdot\boldsymbol{v}\,, \tag{7}$$

and

$$J \simeq 1 + v_r' + 2v_r/r\,. \tag{8}$$

Hence,

$$\frac{\phi(r)}{\phi[s(r)]}(\delta+1) - J \simeq -f^{-1}\nabla_{\boldsymbol{r}}\cdot\boldsymbol{v} - v_r' - \left(2 + \frac{d\ln\phi}{d\ln r}\right)\frac{v_r}{r}\,, \tag{9}$$

and we see that in the series (2), only the term with $n=0$ contributes to linear order. Therefore,

$$\delta_s(\boldsymbol{s}) = \left[-f^{-1}\nabla_{\boldsymbol{r}}\cdot\boldsymbol{v} - v_r' - \left(2 + \frac{d\ln\phi}{d\ln r}\right)\frac{v_r}{r}\right]\Big|_{\tilde{\boldsymbol{r}}=\boldsymbol{s}}\,. \tag{10}$$

This equation is equivalent to equation (3.3) of K87. It also coincides with equation (6) of ND. However, the equation of ND describes the relation between the redshift-space density and the *redshift-space* velocity, $\boldsymbol{u}(\boldsymbol{s})$, and all derivatives are taken there with respect to $\boldsymbol{s}$. The two equations coincide, because

$$\partial u_i/\partial s_j = \partial v_i/\partial r_j + \mathcal{O}\left(v^2\right)\,. \tag{11}$$

Clearly, the equations would be different at the second order.

3.2 Second-order relation

In the series (2), only the terms with $n=0$ and $n=1$ generate the terms up to quadratic in velocity. To second order,

$$\frac{\phi(r)}{\phi[s(r)]} \simeq 1 - D_1 v_r/r + \left(D_1^2 - D_2\right)(v_r/r)^2\,, \tag{12}$$

where

$$D_1 = \frac{d\ln\phi}{d\ln r}\,, \tag{13}$$

and

$$D_2 = \frac{\phi'' r^2}{2\phi}\,. \tag{14}$$

Next,

$$J \simeq 1 + v_r' + 2v_r/r + 2v_r' v_r/r + (v_r/r)^2\,. \tag{15}$$

Finally, the real-space density is up to second order a local function of the two velocity scalars: the expansion (the divergence), θ, and the shear, Σ. Specifically (Chodorowski 1997; see also Gramann 1993, Catelan et al. 1995, Mancinelli & Yahil 1995)

$$\delta(\boldsymbol{r}) = -f^{-1}\theta(\boldsymbol{r}) + \tfrac{4}{21}f^{-2}\left[\theta^2(\boldsymbol{r}) - \tfrac{3}{2}\Sigma^2(\boldsymbol{r})\right]\,. \tag{16}$$

Here,

$$\Sigma^2 \equiv \Sigma_{ij}\Sigma_{ij}\,, \tag{17}$$

$$\Sigma_{ij} \equiv \tfrac{1}{2}\left(\partial v_i/\partial r_j + \partial v_j/\partial r_i\right) - \tfrac{1}{3}\delta^K_{ij}\theta\,, \tag{18}$$

$$\theta \equiv \nabla_{\boldsymbol{r}} \cdot \boldsymbol{v}\,, \tag{19}$$

and I have neglected the weak Ω-dependence. The symbol δ^K_{ij} denotes the Kronecker delta.

From equations (12), (15), and (16) the contribution from the term $n = 0$ in the series (2) can be obtained. As for the term $n = 1$, there is an extra factor v_r in front of the expression in round parentheses $(\phi(r)/\phi(s)\ldots)$. Therefore, it is sufficient to adopt for this expression its linear approximation (9). After a straightforward but rather lengthy algebra we obtain

$$\begin{aligned}\delta_s(\boldsymbol{s}) \quad = \quad & \Big\{-f^{-1}\theta - v_r' - (2+D_1)v_r/r + \left[v_r(f^{-1}\theta + v_r')\right]' + \tfrac{4}{21}f^{-2}\left(\theta^2 - \tfrac{3}{2}\Sigma^2\right) \\ & +(2+D_1)\left(f^{-1}\theta + 2v_r'\right)v_r/r + \left(1 + D_1 + D_1^2 - D_2\right)\left(v_r/r\right)^2\Big\}\Big|_{\tilde{\boldsymbol{r}}=\boldsymbol{s}}\,.\end{aligned} \tag{20}$$

In the DOL, $v_r/r \to 0$ and lines-of-sight are taken at a fixed direction, so $v_r \to v_z$ and $\partial/\partial r \to \partial/\partial z$, and the above formula reduces to equation (13) of C99.

K87 derived a relation between the redshift-space density and the real-space velocity only at the linear order. To derive a related relation, between the redshift-space density and the *redshift*-space velocity, ND applied the Zel'dovich approximation. However, they subsequently restricted their analysis again to the linear regime. The reason they did so was that the redshift-space velocity field is rotational at second order, so that it cannot be reconstructed from radial components of velocities solely. The analysis presented here is free of this problem since we relate the redshift-space density field directly to the real-space velocity field, which is irrotational without any restrictions (before shell crossings in real space).

Applying linear theory in density–velocity comparisons leads to a bias in the inferred value of Ω. We can estimate the amplitude of this bias using equation (20). For simplicity, adopt the DOL and consider a spherical top-hat overdensity. Then

$$\delta_s = -\left(1 + \frac{f}{3}\right)f^{-1}\theta + \left(\frac{4}{21} + \frac{f}{3} + \frac{f^2}{9}\right)f^{-2}\theta^2 \tag{21}$$

(cf. eq. 17 of C99). Assuming the second term on the right-hand side of the above equation to be a small correction it is straigthforward to show that the relative systematic error of Ω is

$$\frac{\Delta\Omega}{\Omega} \simeq \frac{5}{3}\left(\frac{4}{21}\Omega^{-0.6} + \frac{1}{3} + \frac{1}{9}\Omega^{0.6}\right)\theta\,. \tag{22}$$

In the Mark III catalog, the r.m.s. value of θ, when smoothed over scales of 1200 km s^{-1}, is ~ 0.3 (Sigad et al. 1998). Hence, the relative bias of the linear estimate of Ω in *redshift space* is about 40% for $\Omega = 0.3$, and about 30% for $\Omega = 1.0$. This may be compared to the corresponding bias of the linear estimate of Ω in *real space,* about 20 and 10% respectively. (It is obtained by setting the second and the third terms in parentheses in equation 22 equal to zero.) Thus, the nonlinear effects in redshift space are stronger than in real space. Obviously, the bias will be respectively greater for smaller smoothing scales.

Equation (20) can be used to reconstruct the real-space velocity field from the associated redshift-space density field. Since the real-space velocity field is irrotational, it can be described as a gradient of the velocity potential,

$$\boldsymbol{v}(\boldsymbol{r}) = -\nabla_{\boldsymbol{r}}\Phi_v\,. \tag{23}$$

Equation (20) reduces then to a nonlinear differential equation for the velocity potential. As long as the nonlinearities in (20) are weak, an iterative method of solution can be applied. First, we solve the linear part of (20), and the method of ND (the decomposition in spherical harmonics) is fully applicable. Next, we find the second-order solution by solving again the linear equation, with the source term resulting from the density modified by nonlinear contributions approximated by first-order solutions. Specifically,

$$f^{-1}\Delta_{\boldsymbol{r}}\Phi_v^{(2)} + \frac{\partial^2\Phi_v^{(2)}}{\partial r^2} + \frac{2+D_1}{r}\frac{\partial\Phi_v^{(2)}}{\partial r} = \delta_s(\tilde{\boldsymbol{s}} = \boldsymbol{r}) - \mathcal{N}_2\left[\Phi_v^{(1)}(\boldsymbol{r})\right]\,, \tag{24}$$

where $\mathcal{N}_2$ is a sum of all terms quadratic in velocity in equation (20), expressed as functions of the potential.

In this way, from the redshift-space density field we can reconstruct the real-space velocity potential, and hence the real-space velocity field. The latter can then be compared to measured radial velocities of galaxies. Note that this is a velocity–velocity comparison; the application of the 'redshift-real' DVR to density–density comparisons has been discussed in C99.

4 GALAXY DENSITY VERSUS VELOCITY

Galaxies are good tracers of the velocity field induced by the mass distribution. On the other hand, they are biased tracers of the mass distribution itself (see theoretical arguments of Kaiser 1984, Davis et al. 1985, Bardeen et al. 1986, Dekel &

Silk 1986, Cen & Ostriker 1992, Kauffmann, Nusser & Steinmetz 1997 and observational evidence by Davis & Geller 1976, Dressler 1980, Giovanelli, Haynes & Chincarini 1986, Santiago & Strauss 1992, Loveday et al. 1996, Hermit et al. 1996, Guzzo et al. 1997, Giavalisco et al. 1998). In this section I will derive a relation between the redshift-space *galaxy* density field and the real-space velocity field under the assumption of a nonlinear but local bias. This is only a toy model for bias because there are good reasons to believe that bias is in fact somewhat stochastic (Dekel & Lahav 1998, Pen 1998, Tegmark & Peebles 1998, Blanton et al. 1998, Blanton et al. 1999, Tegmark & Bromley 1998). However, it allows for a number of important conclusions to be drawn.

Equation (2) implicitly assumes no bias between the distribution of galaxies and mass, so that the densities appearing in it are in fact the *galaxy* densities $\delta_s^{(\mathrm{g})}$ and $\delta^{(\mathrm{g})}$ (redshift- and real-space galaxy density contrasts respectively). Hence, a more general form of equation (2) is

$$\delta_s^{(\mathrm{g})}(\boldsymbol{s}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left\{ \frac{1}{r^2} \frac{\partial^n}{\partial r^n} \left[r^2 v_r^n \left(\frac{\phi(r)}{\phi[s(r)]} \left[\delta^{(\mathrm{g})} + 1 \right] - J \right) \right] \right\} \Bigg|_{\tilde{\boldsymbol{r}}=\boldsymbol{s}} . \tag{25}$$

In a local bias model, the real-space galaxy density contrast is assumed to be in general a nonlinear function of the mass density contrast (Fry & Gaztañaga 1993; see also Juszkiewicz et al. 1995),

$$\delta^{(\mathrm{g})}(\boldsymbol{r}) = \mathcal{B}\left[\delta(\boldsymbol{r})\right] . \tag{26}$$

Given the real-space DVR (4), equation (25) yields

$$\delta_s^{(\mathrm{g})}(\boldsymbol{s}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left\{ \frac{1}{r^2} \frac{\partial^n}{\partial r^n} \left[r^2 v_r^n \left(\frac{\phi(r)}{\phi[s(r)]} \left[\mathcal{B} \circ \mathcal{F}(\partial v_i/\partial r_j) + 1 \right] - J \right) \right] \right\} \Bigg|_{\tilde{\boldsymbol{r}}=\boldsymbol{s}} . \tag{27}$$

Up to second order, $\mathcal{F}$ is given by expression (16) and

$$\mathcal{B}\left[\delta(\boldsymbol{r})\right] = b\delta(\boldsymbol{r}) + \tfrac{1}{2} b_2 \left[\delta^2(\boldsymbol{r}) - \sigma_\delta^2\right] . \tag{28}$$

Here, b and b_2 are respectively the linear and nonlinear (second-order) bias parameters. The term $\sigma_\delta^2 \equiv \langle \delta^2 \rangle$ ensures that the mean value of $\delta^{(\mathrm{g})}$ vanishes, as required. Combining (28) and (27), and truncating the series at second-order terms we obtain†

$$\begin{aligned} \delta_s^{(\mathrm{g})}(\boldsymbol{s}) \;\; = \;\; & \Bigg\{ -\beta^{-1}\theta - v_r' - (2 + D_1) v_r/r + \left[v_r(\beta^{-1}\theta + v_r') \right]' + \left(\frac{4}{21b} + \frac{b_2}{2b^2} \right) \beta^{-2} \left(\theta^2 - \tfrac{3}{2}\Sigma^2 \right) \\ & + (2 + D_1) \left(\beta^{-1}\theta + 2v_r' \right) v_r/r + \left(1 + D_1 + D_1^2 - D_2 \right) (v_r/r)^2 \Bigg\} \Bigg|_{\tilde{\boldsymbol{r}}=\boldsymbol{s}} . \end{aligned} \tag{29}$$

Expressing velocity in terms of the velocity potential yields

$$\beta^{-1} \Delta_{\boldsymbol{r}} \Phi_v^{(2)} + \frac{\partial^2 \Phi_v^{(2)}}{\partial r^2} + \frac{2 + D_1}{r} \frac{\partial \Phi_v^{(2)}}{\partial r} = \delta_s^{(\mathrm{g})}(\tilde{\boldsymbol{s}} = \boldsymbol{r}) - \mathcal{N}_2 \left[\Phi_v^{(1)}(\boldsymbol{r}), \beta, \beta_2 \right] , \tag{30}$$

where

$$\beta = f(\Omega)/b \tag{31}$$

and

$$\beta_2 = \left(\frac{4}{21b} + \frac{b_2}{2b^2} \right)^{-1} \beta^2 . \tag{32}$$

5 MEASURING Ω AND BIAS SEPARATELY‡

Based on (30), we can reconstruct the real-space velocity field from the associated redshift-space galaxy density field. Comparison of the predicted velocity field to the observed one will yield the best-fit values of the parameters β and β_2. They are a combination of three physical parameters: Ω, b, and b_2. Therefore, we have two constraints for three parameters, so we need an additional constraint on them. As this constraint one can adopt the large-scale galaxy density skewness, which involves b and b_2 (B99).

Galaxy skewness can be measured only in redshift space. Therefore, we need a theoretical relation between the redshift-space galaxy density skewness $S_{3s}^{(\mathrm{g})}$ (which we can measure) and the redshift-space mass density skewness S_{3s} (which we can compute). Approximately this relation is (see C99 for details),

† For simplicity, I have approximated the quantity $\theta^2 - \sigma_\theta^2$ by $\theta^2 - (3/2)\Sigma^2$. This is legitimate, since the mean value of the shear scalar given the velocity divergence, $\langle \Sigma^2 \rangle|_\theta$, is up to second order equal to $\langle \Sigma^2 \rangle = (2/3)\sigma_\theta^2$ (Chodorowski 1997).

‡ This Section is similar to Section 5 of C99. I include it here for the completeness of the present paper.

$$S_{3s}^{(\mathrm{g})} = \frac{S_{3s}}{b} + 3\frac{b_2}{b^2}\,. \tag{33}$$

The redshift-space mass density skewness was computed for scale-free power spectra by Hivon et al. (1995). They found that it depends only very weakly on Ω and that it can be well approximated by the real-space skewness, except in the case of Ω approaching unity *and* the spectral index $n \gtrsim 0$. For more realistic power spectra the calculation can be done in an analogous way, and the result is also expected to be well approximated by the corresponding real-space value.

The redshift-space galaxy density skewness of the *IRAS* density field was measured directly by Bouchet et al. (1993). Kim & Strauss (1998) pointed out that sparse sampling of the *IRAS* galaxies, coupled with the moments method used by Bouchet et al., makes the skewness estimates biased low. However, they proposed a method to measure the skewness by fitting the Edgeworth expansion to the galaxy count probability distribution function around its maximum, properly accounting for the shot noise. Using mock catalogs, Kim & Strauss showed that this estimate of skewness is robust to sparse sampling; they then used this method to measure the skewness of the *IRAS* density field. Hence, we know the value of the skewness of the distribution of the *IRAS* galaxies (and we know an effective means to measure it in other galaxy catalogs).

Equations (31)–(33) are three independent constraints for the parameters Ω, b and b_2, so that they can be used to measure Ω and bias separately. The additional constraint on Ω and bias (the skewness) is to be inferred from the density field alone, making any additional observations unnecessary. In other words, there is enough information in the density field and the corresponding velocity field to break the Ω–bias degeneracy.

6 SUMMARY

I have derived a nonlinear local relation between the *redshift-space* galaxy density field and the *real-space* velocity field. A direct comparison of the redshift-space density directly to the real-space velocity makes the model-dependent reconstruction of the density field in real space unnecessary. Furthermore, the comparison need not to be restricted to the linear regime because the nonlinear real-space velocity field, unlike the redshift-space one, is irrotational.

The analysis presented here is an extension of the work of C99, who performed a similar analysis in the distant observer limit. However, available catalogs of peculiar velocities are too shallow for this limit to be approached (Zaroubi & Hoffman 1996, Szalay, Matsubara & Landy 1998). Here I have relaxed this approximation, and I have taken the radial nature of redshift distortions fully into account. C99 showed how to apply the 'redshift-real' density–velocity relation to the so-called density–density comparisons. Here I have shown how to apply it to velocity–velocity comparisons as well. Namely, I have derived an equation enabling one to reconstruct the real-space velocity field from the redshift-space galaxy density field. I have explicitly derived the density–velocity relation up to linear and second-order terms, but the derivation can be straightforwardly extended to any desired order using the exact equation (27). I have compared the linear relation to that of K87 and of ND and found them to be in agreement.

Applying the second-order relation derived here, or its higher-order extension, to density–velocity comparisons will remove one of the sources of bias (e.g., the nonlinear effects) in the estimate of β, or Ω. I have shown this bias to be substantial in the linear theory. For the fields smoothed over scales as large as 1200 km s^{-1}, the relative systematic error of the linear estimate of Ω is 30–40% (depending on the value of Ω). Moreover, combining the nonlinear relation with an additional measurement (e.g., the skewness of the galaxy density field) breaks the degeneracy between Ω and the galaxy bias.

The ultimate goal of density–velocity comparisons is to estimate the value of Ω separately from the galaxy bias. The method proposed here offers such a possibility, and it will be developped in forthcoming papers. Issues remaining to be addressed are the spatial scales at which a perturbative approach in redshift space can be applied, the stochastic character of bias, and effects of field smoothing. Kudlicki & Chodorowski (in preparation) study the first issue. Taruya & Soda (1998) derived a mildly nonlinear galaxy density–mass density relation in a stochastic bias model. The calculation of the galaxy density–velocity relation in this model will be to some extent similar. Field smoothing affects the real-space density–velocity relation in a very weak way (Chodorowski & Łokas 1997, Chodorowski et al. 1998, Bernardeau et al. 1999). The effects of smoothing are somewhat stronger in redshift space (C99). In any case, they should be studied individually for any particular comparison, because of different smoothing schemes used. For example, in the analysis of ND, both the density and the velocity fields are smoothed, while in the VELMOD analysis of Willick & Strauss (1998), only the *IRAS* density field is smoothed. In accounting for smoothing, the method of C99 may be applied.

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APPENDIX A: DERIVATION OF EQUATION (2)

Here I outline the derivation of the exact equation (2), expressing the redshift-space density field at $\boldsymbol{s}$ in terms of the real-space quantities at $\tilde{\boldsymbol{r}} = \boldsymbol{s}$. The derivation follows to some extent the lines of Appendix A of C99, where a similar equation has been derived in the DOL.

From the conservation of the number of galaxies we have

$$\delta_s(\boldsymbol{s}) = \frac{\phi(r)}{\phi[s(r)]} J^{-1}(\boldsymbol{r})[\delta(\boldsymbol{r}) + 1] - 1 \, , \tag{A1}$$

where ϕ is the selection function and J is the Jacobian of the mapping from real to redshift space. The Fourier transform of the redshift-space density contrast, using the mapping (1), is

$$\delta_s(\boldsymbol{k}) \equiv \int \frac{\mathrm{d}^3 s}{(2\pi)^3} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{s}} \delta_s(\boldsymbol{s}) = \int \frac{\mathrm{d}^3 r}{(2\pi)^3} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \mathrm{e}^{-ik_r v_r(\boldsymbol{r})} \left\{ \frac{\phi(r)}{\phi[s(r)]} [\delta(\boldsymbol{r}) + 1] - J(\boldsymbol{r}) \right\} \tag{A2}$$

(cf. eq. 4 of Scoccimarro et al. 1998 in the DOL). The inverse Fourier transform of the above equation is

$$\delta_s(\boldsymbol{s}) \equiv \int \mathrm{d}^3 k \, \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{s}} \delta_s(\boldsymbol{k}) = \int \frac{\mathrm{d}^3 r \, \mathrm{d}^3 k}{(2\pi)^3} \left\{ \frac{\phi(r)}{\phi[s(r)]} [\delta(\boldsymbol{r}) + 1] - J(\boldsymbol{r}) \right\} \mathrm{e}^{-ik_r v_r(\boldsymbol{r})} \mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{r})} . \tag{A3}$$

Here, I have changed the order of integration. Expanding the first exponent of the integrand we have

$$\mathrm{e}^{-ik_r v_r(\boldsymbol{r})} \mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{r})} = \sum_{n=0}^{\infty} \frac{v_r^n(\boldsymbol{r})}{n!} (-ik_r)^n \mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{r})} = \sum_{n=0}^{\infty} \frac{v_r^n(\boldsymbol{r})}{n!} \frac{\partial^n}{\partial r^n} \mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{r})} , \tag{A4}$$

hence

$$\delta_s(\boldsymbol{s}) = \sum_{n=0}^{\infty} \frac{1}{n!} \int \mathrm{d}^3 r \, v_r^n(\boldsymbol{r}) \left\{ \frac{\phi(r)}{\phi[s(r)]} [\delta(\boldsymbol{r}) + 1] - J(\boldsymbol{r}) \right\} \frac{\partial^n}{\partial r^n} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \mathrm{e}^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{r})} . \tag{A5}$$

The integral over k yields the Dirac delta distribution, $\delta_{\mathrm{D}}(\boldsymbol{s} - \boldsymbol{r})$. Now the following lemma will be useful:

$$\int \mathrm{d}^3 r \, g(\boldsymbol{r}) \frac{\partial^n}{\partial r^n} \delta_{\mathrm{D}}(\tilde{\boldsymbol{r}} - \boldsymbol{r}) = (-1)^n \left. \left\{ \frac{1}{r^2} \frac{\partial^n}{\partial r^n} \left[r^2 g(\boldsymbol{r}) \right] \right\} \right|_{\tilde{\boldsymbol{r}}} , \tag{A6}$$

which can be easily proven by integrating by parts. Using this lemma in equation (A5) yields equation (2).